Ergodic geometry, number theory and Margulis legacy: the next generation

12–15 June, 2022, Chicago, IL

Aaron Brown

Title: Projective actions of lattices in higher-rank Lie groups.

Abstract: Inspired by Margulis's Measurable Factor Theorem, we study smooth actions by lattices in higher-rank Lie groups whose infinitesimal dynamics resembles the dynamics of projective boundary actions. Employing non-uniform measure rigidity and non-stationary normal forms, we establish a number new rigidity results including the following: classification of actions in dimension n by lattices in $\mathrm{SL}(n, \mathbb{R})$, local smooth rigidity of boundary actions, and smooth factor theorems for boundary actions. This is joint work with Federico Rodriguez Hertz and Zhiren Wang.

Nicolas de Saxcé

Title: Diophantine approximation and flows on homogeneous spaces.

Abstract: Following the strategy of S.G. Dani, D.Y. Kleinbock and G.A. Margulis, we shall use diagonal flows on lattice spaces to revisit old problems in diophantine approximation. In particular, this will allow us to answer some questions of W.M. Schmidt on best diophantine exponents for approximation of a real k-dimensional subspace in $\mathbb{R}^d$ by rational subspaces of arbitrary dimension $j = 1, \ldots, d$.

Mikolaj Fraczyk

Title: Growth dichotomies in random environment.

Abstract: It is well known that the number of elements of a subgroup in an R-ball has an exponential growth rate. This observation is, for example, one of the starting points of construction of the Patterson-Sullivan measure for subgroups of hyperbolic groups. It is sometimes useful to consider more general subsets of the group, like for example the connected components of an invariant bond percolation on the Cayley graph. Already for free groups such connected components might fail to have a growth rate, but there is a curious dichotomy where the growth can be shown to exist once the "upper growth" passes certain threshold. If time permits I will speculate on the connection to the growth of non-positively curved measured foliations. Joint work with Miklos Abert and Ben Hayes.

Sebastian Hurtado-Salazar

Title: The thin part of an arithmetic locally symmetric space.

Abstract: We discuss a new arithmetic version of the classical (Zassenhaus or Margulis) lemma regarding the topology of the thin part of a locally symmetric space. We will give several applications including some new results about triangulations and bounds on the homology of arithmetic locally symmetric spaces (joint with Mikolaj Fraczyk and Jean Raimbault).

If time allows us, we will discuss another proof of this new lemma based on the existence of curious word maps known as almost laws (joint with Homin Lee and Lvzhou Chen) and some results regarding the finiteness of arithmetic hyperbolic reflection groups (joint with David Fisher).

Fanny Kassel

Title: Sharpness of proper cocompact actions and applications.

Abstract: We prove the so-called Sharpness Conjecture: any properly discontinuous and cocompact action of a discrete group on a real reductive homogeneous space G/H satisfies a strong form of properness called sharpness. As an application, for G/H of real corank one, such actions are characterized in terms of Anosov representations in the sense of Labourie, and in particular they are stable under small deformations. Sharpness also allows us to deduce the nonexistence of proper cocompact actions on certain homogeneous spaces such as $\mathrm{SL}(2n,\mathbb{R})/\mathrm{SL}(2n-k,\mathbb{R})$ for $k=1$ or 2. Joint work with N. Tholozan.

Asaf Katz

Title: Measure rigidity theorems in smooth dynamics.

Abstract: Classifying the invariant measures for a given dynamical system is a fundamental problem. In the field of homogeneous dynamics, several important theorems give us an essentially complete picture. Moving away from homogeneous dynamics — results are scarcer, mainly due to some profound difficulties carrying out the techniques used in homogeneous dynamics.

A recent development in Teichmuller dynamics — the celebrated magic wand theorem of Eskin–Mirzakhani, gives one such example and actually provides a technique — the factorization method — for proving such results in certain systems.

I will explain how one can implement the factorization method of Eskin–Mirzakhani in smooth dynamics, in order to achieve measure classification of u-Gibbs states for non-integrable Anosov actions. Moreover, I will try to explain some applications of the theorem, including a result of Avila–Crovosier–Eskin–Potrie–Wilkinson–Zhang towards Gogolev's conjecture on actions on the 3D torus.

Osama Khalil

Title: Horocycles in the level aspect and a question of Mahler.

Abstract: We describe recent joint work with Manuel Luethi giving a complete analog of Khintchine's theorem for certain self-similar fractal measures. Our approach is based on linking the problem to the effective distribution of certain random walks on S-arithmetic homogeneous spaces. We will discuss a curious obstacle in the scope of our techniques presented by the volume dependence of the effective rate of equidistribution of horocycles on congruence quotients.

Ilya Khayutin

Title: Class Group Actions: Measure Rigidity, L-functions and Sieving.

Abstract: The linearization method of Dani-Margulis controls the amount of time a unipotent trajectory spends near invariant subvarieties of a homogeneous space. I will describe a problem in number theory where a similar control is desired for diagonalizable torus actions, but no analog of linearization holds. I will present two alternative approaches. The first one draws on measure rigidity for higher rank diagonalizable actions. The second approach is new joint work with V. Blomer and F. Brumley. This method uses the relation between torus periods on GL_2 and L-functions. The two are ostensibly very different, but they share a common underlying structure, and both rely crucially on sieve methods.

Or Landesberg

Title: Horospherical group actions and rigidity of infinite measures in higher rank.

Abstract: Horospherical group actions on homogeneous spaces exhibit remarkable rigidity, as first demonstrated by Furstenberg's proof of unique ergodicity of the horocycle flow on compact hyperbolic

surfaces. Subsequent work by Dani, Veech, Margulis and Ratner led to a complete classification of all finite ergodic measures with respect to such actions. In contrast, much less is known regarding infinite ergodic Radon measures — a natural object to consider in the context of infinite volume homogeneous spaces. In this talk we will describe an infinite measure rigidity result for horospherical group actions on a certain family of homogeneous spaces of higher rank. As a consequence we derive a unique ergodicity type statement for quotients by Zariski dense Anosov subgroups. Based on joint work with Minju Lee, Elon Lindenstrauss and Hee Oh.

Minju Lee

Title: From Circle packings to Torus packings.

Abstract: Let Γ be a self joining of a convex cocompact Kleinian group in $\mathrm{PSL}_2(\mathbb{C})^d$. We obtain counting and equidistribution results for torus packings in $\mathbb{C}^d$ which are invariant under Γ, extending the work of Oh–Shah on the study of circle packings ($d = 1$). This is joint work with Samuel Edwards and Hee Oh.

Nicholas Miller

Title: Superrigidity theorems: old and new.

Abstract: In the 1970s seminal work of Margulis showed that higher rank lattices have superrigid representations, which in particular implies that all such lattices are arithmetic. Since then Gromov–Piatetski-Shapiro and Deligne–Mostow have shown that a similar superrigidity theorem cannot hold for all lattices in the isometry group of real or complex hyperbolic space, i.e., in the rank 1 setting. In this talk, we will recount the work of Margulis for higher rank lattices and then go on to discuss how one can prove certain superrigidity/arithmeticity theorems in the rank 1 setting provided that the lattices satisfy a geometric hypothesis. This is joint work with Bader, Fisher, and Stover.

Wenyu Pan

Title: Exponential mixing of flows for geometrically finite hyperbolic manifolds with cusps.

Abstract: Let $\mathbb{H}^n$ be the hyperbolic n-space and Γ be a geometrically finite discrete subgroup in $\mathrm{Isom}^+(\mathbb{H}^n)$ with parabolic elements. We investigate whether the geodesic flow (resp. the frame flow) over the unit tangent bundle $\mathrm{T}^1(\Gamma\backslash\mathbb{H}^n)$ (resp. the frame bundle $\mathrm{F}(\Gamma\backslash\mathbb{H}^n)$) mixes exponentially. This result has many applications, including prime geodesic theorems, orbit counting, equidistribution, etc. This is based on the joint work with Jialun Li and the joint work with Pratyush Sarkar.

Uri Shapira

Title: Stationary measures on hybrid spaces.

Abstract: Given a homogeneous space $X = G/S$ and a probability measure μ on G, we wish to classify the ergodic μ-stationary measures on X.

This question has been studied extensively, but mostly in two completely different settings:

(Type 1) When the homogeneous space is projective (i.e., embedded in the projective space $P(V)$ of a vector space).

(Type 2) When the homogeneous space is a quotient of a Lie group by a lattice in it.

We suggest a setting which is a hybridization of the two, namely, when the homogeneous space is a bundle over a Type 1 space where the fibers are of Type 2. A prime example of such a space is the space $X(k, n)$ of rank-k discrete subgroups of $\mathbb{R}^n$ identified up to homothety. Here the base of the bundle is the Grassmannian of k-planes and the fibers are copies of $\mathrm{SL}(k, \mathbb{R})/\mathrm{SL}(k, \mathbb{Z})$. A natural conjecture is that under some reasonable assumptions on the measure μ, all the ergodic stationary measures are "homogeneous lifts"

(definitions will be given in the talk). In this talk I will discuss work in progress (joint with Uri Bader and Oliver Sargent) in which we prove special cases of this conjecture.

Wouter Van Limbeek

Title: Commensurators of discrete subgroups of Lie groups.

Abstract: Margulis famously established a dichotomy for the commensurator of irreducible lattices in semisimple Lie groups: It is dense if the lattice is arithmetic, and discrete otherwise. Shalom has asked whether this dichotomy generalizes to all Zariski-dense discrete subgroups. We will explore applications of an answer to this question to seemingly unrelated problems, and give a positive answer for normal subgroups of lattices. This is joint work with David Fisher and Mahan Mj.

Peter Varjú

Title: Exponential mixing of commuting nilmanifold automorphisms.

Abstract: Gorodnik and Spatzier proved that $\mathbb{Z}^l$ actions of ergodic nilmanifold automorphisms are mixing of all orders. They also proved an exponential rate for mixing of order at most 3 for Holder test functions. I will discuss a result extending the exponential rate to mixing of all orders. This problem is intimately related to a problem in Diophantine approximation, which is solved using Schmidt's subspace theorem. Joint work with Timothée Bénard.

Zhiren Wang

Title: Effective equidistribution of some one-parameter unipotent flows with polynomial rates.

Abstract: It has been a challenging task to effectivize Ratner's equidisribution theorem beyond horospheric actions. In particular, if G is semisimple and U is a non-horospheric Ad-unipotent subgroup, no quantitative form of Ratner's equidistribution theorem was known with any error rate, though there has been progress by Lindenstrauss–Margulis and Lindenstrauss–Margulis–Mohammadi–Shah on understanding quantitatively density properties of such flows with iterative logarithm error rates. Recently, Lindenstrauss–Mohammadi obtained quantitative orbit density estimates with polynomial rates for Borel subgroups of groups isomorphic to $\mathrm{SL}_2(\mathbb{R})$ in quotients of $\mathrm{SL}_2(\mathbb{C})$ and $\mathrm{SL}_2(\mathbb{R}) \times \mathrm{SL}_2(\mathbb{R})$. In this talk, we will report a joint work with Lindenstrauss and Mohammadi on a fully quantitative and effective equidistribution result in these cases.

Amie Wilkinson

Title: Centralizer rigidity.

Abstract: I will discuss the goals and some highlights of a program with Danijela Damjanović and Disheng Xu to address the question: what can you say about a diffeomorphism that has an unusually large group of symmetries?

Pengyu Yang

Title: Equidistribution of translates of curves in homogeneous spaces and its applications.

Abstract: We study equidistribution problem for translates of curves under 1-parameter diagonal subgroups in homogeneous spaces, and describe geometric/arithmetic obstructions to equidistribution. We will also discuss applications to Dirichlet improvability problem in Diophantine approximation.